Real Options Valuation within Information Uncertainty: Some Extensions and New Results

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ABSTRACT

This paper develops some results regarding the economic value added and real options. We use Merton's (1987) model of capital market equilibrium with incomplete information to introduce information costs in the pricing of real assets. This model allows a new definition of the cost of capital in the presence of information uncertainty. Using the methodology in Bellalah (2001, 2002) for the pricing of real options, we extend the standard models to account for shadow costs of incomplete information.

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I.   INTRODUCTION

Over the last two decades, a body of academic research takes the methodology used in financial option pricing and applies it to real options in what is well known as real options theory. This approach recognizes the importance of flexibility in business activities. Today, options are worth more than ever because of the new realities of the actual economy: information intensity, instantaneous communications, high volatility, etc.

Financial models based on complete information might be inadequate to capture the complexity of rationality in action. As shown in Merton (1987), the "true" discounting rate for future risky cash flows must be coherent with his simple model of capital market equilibrium with incomplete information. This model can be used in the valuation of real assets[1].

Managers are interested not only in real options, but also in the latest outgrowth in DCF analysis; the Economic Value Added. EVA simply means that the company is earning more than its cost of capital on its projects. EVA is powerful in focusing senior management attention on shareholder value. Its main message concerns whether the company is earning more than the cost of capital. It says nothing about the future and on the way the companies can capitalize on different contingencies. Hence, a useful criterion must account for both the DCF analysis and real options. The NPV and the EP (economic profit) ignore the complex decision process in capital investment. In fact, business decisions are in general implemented through deferral, abandonment, expansion or in series of stages. This paper accounts for the effects of information costs in the valuation of derivatives as in Bellalah (2001).

The structure of the paper is as follows. Section II presents a simple framework for the valuation of the firm and its assets using the concept of economic value added in the presence of information costs. Section III develops a simple analysis for the valuation of real options within information uncertainty. Section IV develops a context for the pricing of real options in a continuous-time setting using Standard and complex options. In particular, we extend the model in Triantis and Hodder (1990) for the valuation of flexibility as a complex option within information uncertainty. Section V develops some simple models for the pricing of real options in a discrete time setting by accounting for the role of shadow costs of incomplete information. We first extend the Cox, Ross and Rubinstein (1979) model to account for information costs. Then, we use the generalization in Trigeorgis (1990) for the pricing of several complex investment opportunities with embedded real options.

II.   FIRM VALUATION UNDER INCOMPLETE INFORMATION

We remind first Merton's (1987) model and the definition of the shadow costs of incomplete information.

A.   Merton's model

Merton's model may be stated as follows:

$$\overline{Rs} - R = \beta_s[\overline{R}_m - R] + \lambda_s - \beta_s\lambda_m$$

where $\overline{R}_s$: the equilibrium expected return on security S; $\overline{R}_m$: the equilibrium expected return on the market portfolio; R: one plus the riskless rate of interest, r; $\beta = \dfrac{cov(\overline{Rs}/\overline{Rm})}{Var(\overline{Rm})}$: The beta of security S; λ_s: the equilibrium aggregate "shadow cost" for the security S ; λ_m: the weighted average shadow cost of incomplete information over all securities in the market place.

The CAPM of Merton (1987), referred to as the CAPMI is an extension of the standard CAPM to a context of incomplete information. When $\lambda_m = \lambda_{s = 0,}$ this model reduces to the standard CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966).

B.   Economic Value Added, EVA, and Information Costs

In standard financial theory, every company's ultimate aim is to maximize shareholders' wealth. The maximization of value is equivalent to the maximization of long-term yield on shareholders' investment. Currently, EVA is the most popular Value based measure.

A manager accepts a project with positive NPV; i.e; for which the internal rate of return IRR is higher than the cost of capital. With practical performance measuring, the rate of return to capital is used because the IRR can not be measured. However, the accounting rate of return is not an accurate estimate of the true rate of return. As shown in several studies, ROI underestimates the IRR in the beginning of the period and overestimates it at the end. This phenomenon is known as wrong periodizing.

The EVA valuation technique provides the true value of the firm regardless of how the accounting is done. The EVA is a simply a modified version of the standard DCF analysis in a context where all of the adjustments in the EVA to the DCF must result net to zero.

EVA can be superior to accounting profits in the measurement of value creation. In fact, EVA recognizes the cost of capital and, the riskiness of the company. Maximizing EVA can be set as a target while maximizing an accounting profit or accounting rate of return can lead to an undesired outcome. The weighted average cost of capital, WACC, is computed using Merton's (1987) model of capital market equilibrium with incomplete information for the cost of equity component. The WACC is computed using the CAPMI. Stewart (1990) defines the EVA as the difference between the Net operating profit after taxes (NOPAT) and the cost of capital. EVA gives the same results as the discounted cash flow techniques or the Net present value (NPV). It can be described by one of the three equivalent formulas:

$$EVA = NOPAT - \text{Cost of capital x (Capital employed)}$$

or

$$EVA = \text{Rate of return - Cost of capital x (capital employed)}$$

or

$$EVA = (ROI - WACC) \text{ Capital employed}$$

with   Rate of return = NOPAT/Capital,

where Capital = Total balance sheet - non-interest bearing debt at the beginning of the year. ROI = the return on investment after taxes, i.e; an accounting rate of return.

The cost of capital is the WACC as in the Modigliani-Miller analysis where the cost of equity is defined with respect to the CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966). In the presence of information costs, the cost of capital can be determined in the context of Merton's model of capital market equilibrium as described above. In this case, the above formulas must be used. Hence, the analysis in Stewart (1990) can be extended using the CAPMI of Merton (1987) rather than the standard CAPM in the computation of EVA. In the presence of taxes, EVA can also be calculated as:

EVA = [NOP - ((NOP - Excess depreciation - Other increase in reserves) x (Tax rate))]
-WACC x (Capital)

where NOP is the Net operating profit.

Stewart (1990) defines the Market Value Added, MVA, as the difference between a company's market and book values:

MVA = Total market asset value - Capital invested

When the book and the market values of debt are equal, MVA can be written as:

MVA = Market value of equity - Book value of equity.

The MVA can also be defined as:

MVA = the present value of all future EVA.

Using the above definitions, it is evident that:

Market value of equity = Book value of equity + Present value of all future EVA.

In this context, this formula is always equivalent to discounted cash flow and Net present value. Again, the cost of capital with information costs represents an appropriate rate for the discounting of all the future EVA. Hence, the main concepts in Stewart (1990) can be extended without difficulties to account for the shadow costs of incomplete information in the spirit of Merton's model.

C.   The cost of capital, the firm's value and Information costs

The cost of capital or the weighted average cost of capital, (WACC), is a central concept in corporate finance. It is used in the computation of the Net present value, NPV, and in the discounting of future risky streams. The standard analysis in Modigliani-Miller (1958, 1963) ignores the presence of market frictions and assumes that information is costless. Or, as it is well known in practice, information costs represent a significant component in the determination of returns from investments in financial and real assets.

Merton (1987) provides a simple context to account for these costs by discounting future risky cash flows at a rate that accounts for these costs. In this

context, the cost of capital and the firm's value can be computed in an economy similar to that in Merton (1987). We denote respectively by:

D: the face value of debt,
B: the market value of debt,
S: the market value of equity,
O: perpetual operating earnings,
τ : the corporate tax rate,
Vu: the value of the unlevered firm,
V: the value of the levered firm,
k_d : the cost of debt,
k_b : the current market yield on the debt,
k_e : the cost of equity or the required return for levered equity,
k_o : the market value-weighted of these components known as the WACC,
ρ: the market cost of equity for an unlevered firm in the presence of incomplete information.

Table 1
Summary of the main results regarding the components of the costs of capital and the values of the levered and unlevered firms with information uncertainty

No tax	Corporate tax
$\rho = \dfrac{O}{Su} + \lambda u$	$\rho = (\dfrac{O}{Su} + \lambda u)(1-\tau)$
$B = D\dfrac{k'_d}{k_p}$	$B = D\dfrac{k'_d}{k_b}$
$K_e = \dfrac{[O - k'_d D]}{S}$	$K_e = \dfrac{[(O - k'_d D)(1 - t)]}{S}$
$K_e = \rho + \dfrac{B}{S}(\rho - k'_b)$	$[\rho + \dfrac{B}{S}(\rho - k'_b)](1-\tau)$
$Vu = \dfrac{O}{P} = Su$	$Vu = (1-\tau)\dfrac{O}{p}$
$V = Vu$	$V = Vu + \tau B$
$K_0 = K_e \dfrac{S}{V} + k'_b \dfrac{B}{V}$	$K_0 = K_e \dfrac{S}{V} + k'_b (1-\tau) \dfrac{B}{V}$
$k_0 = \dfrac{o}{v}$	$k_0 = \dfrac{o}{v}(1-\tau)$
$k_0 = \rho$	$k_0 = \rho(1-\tau)\dfrac{B}{V}$

with $k'_b = k_b + \lambda_d$ and $k'_d = k_d + \lambda_d$.

Using the main results in the Modigliani and Miller analysis and Merton's λ, it is clear that discounting factors must account for the shadow cost of information regarding the firm and its assets. By adding Merton's λ in the analysis of Modigliani Miller in the discounting of the different streams of cash-flows for levered and unlevered firms, similar very simple formulas can be derived in an extended Modigliani-Miller-Merton context. The formulas follow directly from the analysis in Modigliani-Miller and the fact that future risky streams must be discounted at a rate that accounts for Merton's λ. The above Table presents the main results regarding the components of the costs of capital and the values of the levered and unlevered firms with information costs.

The term λ_d indicates the information cost for the debt and the term λu corresponds to the information cost for the unlevered firm.

These results show the components of cost of capital and the values of the firms in the presence of information costs. When these costs are equal to zero, this Table is equivalent to the results in the Modigliani-Miller analysis.
The results show how to calculate the firm's value, the weighted average cost of capital, and the Net present value of future risky cash flows in the presence of information costs.

The above formulas are simulated for an illustrative purpose using: $O = 2\,000$, $D = 10\,000$, $B = 10\,000$, $S = 10\,000$, $V = 20\,000$, $\tau = 40\%$, $\rho = 10\%$ and $k_d = 5\%$, $\lambda u = 0\%$, $\lambda_d = 0\%$. These figures represent the standard benchmark case. The simulations allow appreciating the impact of information costs on the computation of the different values of the levered and unlevered firm and the costs of capital with corporate taxes.

Table 2
Summary of the main results regarding the components of the costs of capital and the values of the levered and unlevered firms with information costs: the standard case. $O = 2000$, $D = 10\,000$, $B = 10\,000$, $S = 10\,000$, $V = 20\,000$, $\tau = 40\%$, $\rho = 10\%$ and $k_d = 5\%$, $\lambda_u = 0\%$, $\lambda_d = 0\%$.

No tax	Corporate tax
$\rho = 10\%$	$\rho = 10\%$
$B = 10000$	$B = 10000$
$k_e = 15\%$	$k_e = 15\%$
$k_e = 15\%$	$k_e = 15\%$
$Vu = 20000$	$Vu = 20000$
$V = 20000$	$V = 16000$
$S = 10000$	$S = 6000$
$k_0 = 10\%$	$k_0 = 7.5\%$
$k_0 = 10\%$	$k_0 = 7.50\%$
$k_0 = 10\%$	$k_0 = 7.5\%$

The fact that k_e is equal to 15% in this case is consistent with the MM assumptions. The effect of incomplete information on the firm value and the cost of capital is simulated using the following data: O = 2000, D = 10 000, B = 10 000, S = 10 000, V = 20 000, τ = 40 %, ρ= 10 %, k_d = 5 %, λu = 3 %, λ_d = 1 %.

Table 3
The main results for the cost of capital and the values of
the levered and unlevered firms with information costs
O = 2000, D = 10 000, B = 10 000, S = 10 000, V = 20 000, τ = 40%, ρ = 10%,
k_d =5%, λ_u = 3%, λ_d = 1%

No tax	Corporate tax
ρ = 13%	ρ = 13%
B = 10000	B = 10000
k_e = 26%	k_e = 26%
k_e = 26%	k_e = 26%
Vu = 15384.62	Vu = 9230.77
Vu = 15384.62	Vu = 13230.77
S = 5384.62	S = 3230.77
k_0 = 13%	k_0 = 9.07%
k_0 = 13%	k_0 = 9.07%
k_0 = 13%	k_0 = 9.07%

The value of k_e is equal to 26 % in this case. Every scenario is consistent with the Modigliani-Miller assumptions and the Merton's shadow cost (λ). When compared to the benchmark case with no information costs, we see that information costs increase significantly k_e. These shadow costs reduce the value of the firm in the two cases: with no tax and with corporate tax.

II.   THE VALUATION OF REAL OPTIONS WITH INFORMATION COSTS IN A CONTINUOUS-TIME SETTING

Several models in financial economics are proposed to deal with the ability to delay irreversible investment expenditure[2]. Information costs are used in the valuation of real options in Bellalah (2001, 2002).

A.   The pricing of derivatives in the presence of information costs

As in Bellalah (2001), let's denote by C the price of a derivative security on a stock With a continuous dividend yield δ. The underlying asset price dynamics are:

$$dS = \mu Sdt + \sigma S\, dz$$

where the drift term μ and the volatility σ are constants and dz is a Wiener process.

Using Ito's lemma, we have:

$$dC = (\frac{\partial C}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2)dt + \frac{\partial C}{\partial S} \sigma S dz$$

We construct a portfolio Π using a position in the derivative security and a number of units of the underlying asset $\Pi = - C + \frac{\partial C}{\partial S} S$. The change in the portfolio value is $\Delta\Pi = (-\frac{\partial C}{\partial S} \mu S - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2) \Delta t$. Over the same time interval, dividends are given by $\delta S \frac{\partial C}{\partial S} \Delta t$. Let us denote by ΔW the change in the wealth of the portfolio holder. We have

$$\Delta W = (-\frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \delta S \frac{\partial C}{dS})\Delta t$$

The portfolio is instantaneously risk-less and must earn the risk-free rate plus information costs or

$$(-\frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \delta S \frac{\partial C}{dS})\Delta t = -(r + \lambda c)C\Delta t + (r + \lambda s)S \frac{\partial C}{dS})\Delta t$$

where λ_i refers to these costs. This gives

$$\frac{\partial C}{\partial t} + (r + \lambda s - \delta) S \frac{\partial C}{\partial S} \Delta t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 = (r + \lambda c)C$$

Bellalah (1999) provides the following equation for the pricing of commodity options:

$$\frac{1}{2} \sigma^2 S^2 C_{ss} + (b + \lambda_s) SC_s - (r + \lambda_c)C + C_t = 0$$

When the information costs λs and λc are set equal to zero, this equation collapses to that in Barone-Adesi and Whaley (1987). The term b indicates the cost of carrying the commodity. The value of a European commodity call is:

$$C(S,T) = Se^{((b-r-(\lambda c - \lambda s))T)} N(d_1) - Ke^{-(r+\lambda c)T} N(d_2)$$

with
$$d_1 = [\ln(\frac{S}{K}) + (b + \frac{1}{2} \sigma^2 + \lambda s)T]/ \sigma \sqrt{T}$$

where N(.) is the cumulative normal density function.

When λs and λc are equal to zero and b = r, this formula is the same as that in Black and Scholes. A direct application of the approach in Barone-Adesi and Whaley (1987), allows writing down immediately the formulas for American commodity options with information costs. The following Tables provide simulations results regarding our model with incomplete information and the Black and Scholes model. Option values are compared for different levels of the underlying asset (from 70 to 120) and different information costs regarding the option and its underlying asset.

Table 4
Call options values using the following parameters:
K =100, r=0.08, t=0.25, σ =0.2

Black & Scholes		Incomplete Information			(λs, λc)		
		.01,.001	.01 , 0	.03,.001	.03 , 0	.2 ,.01	.2 ,0
S:70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
S:80	.0000	.0000	.0000	.0000	.0000	.0000	.0000
S:90	0.8972	0.9009	0.8949	0.8964	0.8904	0.8533.	0.8534
S:100	5.0177	5.0202	5.6705	7.6257	4.9802	4.7872.	4.7730
S:110	12.6520	12.6448	12.6204	12.5817	12.5574	12.058	12.034
S:120	22.0877	22.0619	22.0326	21.9586	21.9227	21.038	21.010

Table 5
European Futures Call values using the following parameters:
K =100, r=0.08, T=0.5, σ =0.4

Complete Information Black's model		Incomplete Information model	
Futures price	Option price	λc = 1%	λc = 5%
F = 70	0.0000	0.0000	0.0000
F = 80	2.6940	2.6806	2.6275
F = 90	6.1331	6.1025	5.9817
F = 100	10.8051	10.7512	10.5383
F = 110	16.7917	16.7260	16.3772
F = 120	19.9557	23.7260	23.3058

B.   Investment timing, project valuation and the pricing of real assets with compound options within information uncertainty.

The timing option gives the right to the manager to choose the most advantageous moment to implement the investment project and allows him to pull out of the project when the economic environment turns out to be unfavourable. Several standard models are proposed in the literature for the pricing of these options.

Lee (1988) proposes a model for the valuation of the timing option arising from the uncertainty of the project value and for the detection of the optimal timing. He

considers three cases: the optimal timing of plant and equipment replacement, the real estate development and the marketing of a new product.

The investment project is interpreted as the replacement of a capital asset, the inauguration of a new product and the development of real estate. The manager has the option to implement the project in the time interval [0, T] where T is the option's maturity. The possibility to implement an investment project in [0, T] can be seen as an American call option on a security with no dividend payments.

Let us denote by:

V : the present value of the project implemented,

S : the present value of the project not yet implemented,

I : the cost of the project,

D : a known anticipated jump in the project's value,

C(S,0, T, I): an American call without dividend where 0 refers to the starting time,

c(S, 0, T, I) : a European call option,

$P_{Ti}(0, T)$: the value of timing option.

The value of $P_{Ti}(0,T)$ corresponds to the difference between the value of the deferrable investment opportunity when the timing option is "alive" and when the timing option is "dead". The project's value if it is implemented now is:

$$C(S, 0, 0, I) = Max [V - I, 0]$$

where the NPV of the implemented investment opportunity is (V - I).

In this case, the timing option value is given by:

$$P_{Ti}(0,T) = C(S,0,T,I) - C(S,0,0,I)$$
$$P_{Ti}(0, T) = min[C(S,0,T,I), C(S,0,T,I) - (V - I)] \geq 0$$

This equation shows that it is profitable to implement the project now (V - I > 0) when the value of the timing option is equal to the value of the deferrable investment opportunity minus NPV.

The cost of waiting D can be seen as a dividend in the pricing of American call options. It is possible to study three different specifications.

Specification 1:

(i)   The present value changes of the not-yet-implemented project is:

$$dS/S = \mu dt + \sigma dz$$

(ii)   If the project is implemented before t*, it generates an extra cash-flow at t*:

$$V_{t^*} = S_t^* + D$$

This specification corresponds for example to the real estate development. In fact, leaving property vacant can be seen as holding a timing option on the real estate development. The cost of development is I.

Specification 2:

Same as (i) of specification 1. The cost of the project increases by D when implemented after t*:

$$I_{t^* + h} = I \qquad \text{for all } h > 0$$
$$X_{t^* - h} = I - D \quad \text{for } h > 0$$

It is possible to use the formula in Whaley (1981) to compute the value of the optimal timing option and the optimal timing of project implementation. It is possible to show that the value of an American call in the presence of a cash discrete dividend and information costs is given by:

$$C = S[e^{((b-r-(\lambda c - \lambda s))t^*)} N(b_1) + e^{((b-r-(\lambda c - \lambda s))t^*)} N_2 (a_1, -b_1, -\frac{t^*}{T})] - I[e^{-(r+\lambda c)t^*} N(b_2)$$

$$+ e^{-(r+\lambda c)t^*} N2 (a_2, -b_2, - \sqrt{\frac{t^*}{T}})] + De^{-(r+\lambda c)t^*} N(b_2).$$

with :

$$a_1 = [\ln(S/I) + (b+ \frac{1}{2} \sigma^2 + \lambda s)t^*]/ \sigma \sqrt{t^*}$$

$$a_2 = a_1 - \sigma \sqrt{t^*}$$

$$b_1 = [\ln(S/S_{cr,t^*}) + (b+ \frac{1}{2} \sigma^2 + \lambda s)t_i]/ \sigma \sqrt{t^*}$$

$$b_2 = b_1 - \sigma \sqrt{t^*}$$

where S_{cr,t^*} corresponds to the trigger point present value, N(.) stands for the cumulative normal distribution and N_2 (.,.,) is the bivariate cumulative normal density function with upper integral limits a and b and a correlation coefficient ρ^3.
The "trigger point" for specification 1 is given by:

$$P_{Ti} (t^*, T) = c (S_{cr,t^*}, t^*, T, I) - (S_{cr,t^*} + D - I) = 0$$

This case fits well with the replacement of plant and equipment. If we denote by S, I and T the present value, the cost of replacement and the remaining life, then a firm keeping the equipment in operation will face expenditures at time t* of amount D. In this case, formula (3) can be applied to compute the value of the timing option and trigger point present value. These two specifications allow a single occurrence of discrete cash flow at time t*. It is possible to generalize the results using specification 3.

Formula (3) is simulated in the following Tables 6, 7 and 8. The parameters are S = 175, D = 1.5, r = 0.1 and the constant "carrying cost" is 0.6. We use different values for the information costs λ_S and λ_C. The option has a maturity date of one month. The volatility is σ =0.32 and the "dividend" is paid in 24 days.

Table 6 uses these parameters with no information costs. It gives the computation of the American call value referred to as Call, the option c_a, the option c_b, the option c_c, the algebraic sum of the three options ($c_a + c_b - c_c$) and the critical underlying asset price. The results are given for different "strike prices" varying from 100 to 240. Table 6 shows that the algebraic sum of the three options is equal to the American call price. The "critical asset price" corresponding to an early exercise is an increasing function of the strike price.

Table 7 uses the same data except for information costs. Information costs are set equal to $\lambda_s = 0.01$ and $\lambda c = 0.001$. The reader can check that the algebraic sum of the three options is exactly equal to the American call price. With these costs, the call price is slightly higher than in Table 6.

Table 8 uses the same parameters except for the information costs which are set equal to $\lambda_s = 0.1$ and $\lambda c = 0.05$.

Table 6
Simulations of option values for the continuous-time model using parameters:
S = 175, r = 0,1, D = 1,5, T = 30, t = 24, σ = 0,32, λ_c = 0, λs= 0.

Strike	Call	c_a	c_b	c_c	$c_a + c_b - c_c$	S*
100	76.03	74.42	74.25	72.65	76.03	100.02
105	71.06	69.46	69.11	67.51	71.06	105.00
110	66.09	66.09	66.09	66.09	66.09	110.00
115	61.13	61.13	61.13	61.13	61.13	115.00
120	56.16	56.16	56.16	56.16	56.16	120.00
125	51.19	51.19	51.19	51.19	51.19	125.00
130	46.22	46.22	46.22	46.22	46.22	130.00
135	36.30	36.30	36.30	36.30	36.30	135.00
140	31.37	31.37	31.37	31.37	31.37	140.00
145	26.50	26.50	26.50	26.50	26.50	145.00
150	21.78	21.78	21.78	21.78	21.78	150.00
155	17.31	17.31	17.31	17.31	17.31	154.00
160	13.25	13.25	13.25	13.25	13.25	159.00
165	9.72	9.72	9.72	9.72	9.72	164.00
170	6.82	6.82	6.82	6.82	6.82	169.00
175	4.56	4.27	2.52	2.52	2.23	179.00
180	4.56	4.56	4.56	4.56	4.56	179.00
185	2.91	2.74	1.41	1.24	2.91	184.00
190	1.77	1.68	0.74	0.64	1.77	189.00
195	1.03	0.98	0.36	0.31	1.03	194.00
200	0.57	0.55	0.16	0.14	0.57	200.00
240	0.00	0.00	0.00	0.00	0.00	240.00

Table 7
Simulations of option values for the continuous-time model using parameters:
S = 175, r = 0, 1, D = 1,5, T = 30, t = 24, σ = 0,32, λc = 0,001, λs= 0,01.

Strike	Call	c_a	c_b	c_c	$c_a + c_b - c_c$	S*
100	76.03	74.42	74.25	72.65	76.03	100.02
120	71.06	69.46	69.11	67.51	71.06	105.00
140	66.09	66.09	66.09	66.09	66.09	110.00
145	31.37	31.37	31.37	31.37	31.37	140.00
150	61.13	61.13	61.13	61.13	61.13	115.00
155	56.16	56.16	56.16	56.16	56.16	120.00
160	51.19	51.19	51.19	51.19	51.19	125.00
165	46.22	46.22	46.22	46.22	46.22	130.00
170	36.30	36.30	36.30	36.30	36.30	135.00
175	31.37	31.37	31.37	31.37	31.37	140.00
180	26.50	26.50	26.50	26.50	26.50	145.00
185	21.78	21.78	21.78	21.78	21.78	150.00
190	17.31	17.31	17.31	17.31	17.31	154.00
195	13.25	13.25	13.25	13.25	13.25	159.00
200	9.72	9.72	9.72	9.72	9.72	164.00
240	6.82	6.82	6.82	6.82	6.82	169.00

Table 8
Simulations of option values for the continuous-time model using parameters:
S = 175, r = 0,1, D = 1,5, T = 30, t = 24, σ = 0,32, λc = 0,05, λs = 0,1

Strike	Call	c_a	c_b	c_c	$c_a + c_b - c_c$	S*
100	76.03	75.55	74.25	73.58	76.92	100.02
120	57.12	55.79	69.11	67.51	57.12	120.00
140	37.32	36.06.	66.09	66.09	66.09	110.00
145	32.40	31.37	31.37	31.37	31.37	140.00
150	27.54	61.13	61.13	61.13	61.13	115.00
155	22.80	21.71	18.55	56.16	56.16	120.00
160	18.29	51.19	51.19	51.19	51.19	125.00
165	14.15	46.22	46.22	46.22	46.22	130.00
170	10.52	36.30	36.30	36.30	36.30	135.00
175	7.49	31.37	31.37	31.37	31.37	140.00
180	5.09	26.50	26.50	26.50	26.50	145.00
185	3.31	21.78	21.78	21.78	21.78	150.00
190	2.05	17.31	17.31	17.31	17.31	154.00
195	1.22	13.25	13.25	13.25	1.22	194.00
200	0.69	9.72	9.72	9.72	0.69.	200.00
240	0.00	0.00	0.00	0.00	0.00	240.00

Specification 3:

(i)   The present value of the implemented project V follows the equation:

$$dV/V = \mu dt + \sigma dz$$

(ii)   If the project is not implemented immediately, its value will fall by a known amount D_i at time t_i where $i = 1, 2, ..., n$.

(iii)   If the project is implemented at time t_k, its present value is given by:

$$S_k = V_0 - \sum_{i=1}^{k-1} D_i\, e^{-(r+)t_i},\, 0 \qquad t_i < t_k < tT$$

In this expression, S_k corresponds to the present value at time 0 for the project to be implemented at t_k. V_0 corresponds to the present value of the project to be implemented now. The cost of waiting is given by the difference between the two present values. In this case, an extended version of the Black's (1976) approximation with information costs can be use:

$$C(S, 0, T, I) = max[c(Sk, 0, t_k, I) \mid k = 1, 2,..., n]$$

At each instant t_h, just before the known present value decline, D_h, it is Possible to compute the trigger point project value, $V_{cr,h}$ as in Lee (1988) using the following equation :

$$S_k = V_{cr,h} - \sum_{i=1}^{k-1} D_i\, e^{-(r+)\,(t_i - t_h)} \quad k = h, h+1,..., n$$
$$c(S_k, t_h, t_{k*}, I) = V_{cr,h} - I$$

In this expression, t_{k*}: the planned optimal timing when the manager decides to wait; S_{k*}: the present value at t_k of the project when it is implemented at the optimal planned time. A firm has a timing option on the introduction of a product with a cost I for a time horizon T. If a new product is introduced at time 0, its present value V can be described by the above dynamics. Before a given firm introduces the product, the introduction by the competitor at time t_k can reduce the value of a given firm new product by D_k.

Each episode of innovation at time i can reduce the value of the new planned product line by D_i. This fits with specification 3.

III.   VALUING FLEXIBILITY AS A COMPLEX OPTION WITHIN INFORMATION UNCERTAINTY

Triantis and Hodder (1990) propose an approach for the valuation of flexible production systems using the option pricing theory. Their analysis concerns mainly the switching between different operating states in the lines of Majd and Pindyck (1987), McDonald and Siegel (1984, 1986), Pindyck (1988), Brennan and Schwartz (1985), etc. They propose a model for the pricing of complex options that appear in the valuation of a flexible production system. The system allows the manager to switch the output mix

over time. The model accounts for the fact that real assets markets are monopolistic or oligopolistic by allowing downward sloping demand curves for the underlying assets [4].

Consider the decision to purchase a production facility that can be costlessly switched to produce some combinations of k products. The average variable cost of product i is $C_i(t)$ and its sale price is $P_i(t)$. Hence, the per unit profit is $R_i(t) = P_i(t) - C_i(t)$. Using a production rate $q_i(t)$, the profit can be written as a linear function:

$$R_i(t) = A_i - B_i(t) q_i(t)$$

where B_i is a positive constant and A_i satisfies the equation:

$$dA_i = \alpha_i \, dt + \sigma_i dz_i; \quad A_i(0) = a_i; \quad i=1,2,3...k$$

where z_i is the standard Brownian motion with $\rho_{i,j}$ a correlation coefficient between the processes for products i and j.

The life of the production system [0, T] is split up into N periods of equal duration τ. At each period, a production rate $q_i(\tau)$ for the ith product during the period $[n\tau, (n+1)\tau]$ is chosen to define the firm's manufacturing program.

The cost of purchasing a system of capacity Q is $I_0(Q)$. A fixed cost $CF(Q)$ per unit time is suffered regardless of whether the system is operating. It is assumed that portfolios of securities can be constructed and their price processes M_i are given by:

$$\frac{dM_i}{M_i} = \mu_{Mi} dt + \sigma_i dz_i, \quad i = 1, 2, 3...k$$

There exists also a riskless asset D such that $dD_t = r \, D_t \, dt$ where r is the riskless rate. The value of the option to produce in $[n\tau, (n+1)\tau]$, expiring at time $n\tau$ is

$$W^{n\tau}(A_1(t), ..., A_k(t),t) \text{ at } t < n\tau.$$

The option value $W^{n\tau}$ satisfies the following partial differential equation:

$$\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{K}\rho_{ij}\sigma_i\sigma_j \, W^{n\tau} + \sum_{i=1}^{k}(r - \delta_i)W_i^{n\tau} - rW^{n\tau} + W_t^{n\tau} = 0$$

where $\delta_i = \mu_{Mi} - \sigma_i$ for i = 1 to k and the subscripts on W^{nr} refer to partial derivatives. The terminal boundary condition for the option value is:

$$W^{nr}(A_1......A_k, nr) = \sum_{i=1}^{k}q_i(A_i\beta + \eta_i - \beta_i q_i\beta)$$

where

$$\beta = \frac{(1 - e^{-nt})}{r}, \quad \eta_i = \frac{r - \delta_i}{r}[\beta - \tau e^{-rr}], \quad i=1,2,3...k$$

In order to obtain the maxi mum values functions of A_i, for $i = 1, ...,k$, the following quadratic programming problem gives the optimal production levels q_i^*:

$$\max q_i \quad \sum_{i=1}^{k} q_i(A_i\beta + \eta_i - B_i q_i\beta)$$

subject to

$$q_i \geq 0, \quad i=1,2,3...k$$
$$\sum_{i=1}^{k}\frac{q_i}{y_i} - Q = 0$$

Denoting by Γ_i the Lagrange multipliers, the optimal production quantities are determined using the following Kuhn-Tucker conditions:

$$A_i\beta + \eta_i - 2B_i q_i^*\beta - \Gamma_i + \frac{\Gamma k + 1}{y}$$

The complementary slackness conditions are:

$$\Gamma_i q_i = 0; \quad i=1,2,3...k$$
$$\Gamma_k + 1\left[\sum_{i}^{k}\frac{q_i}{y_i} - Q\right] = 0 \text{ }^5$$

To determine the option value $W^{nr}(a_1, a_2, 0)$, Triantis and Hodder(1990) use the Harrison and Kreps (1979) approach and denote by $f^*(A1, A2)$ the transformed probability density over which the expected terminal value of the option is calculated. The transformed bivariate density has means

$$\mu_i = a_i + (r - \delta_i)nr \text{ and variances } \sigma^2_i nr \text{ for } i = 1,2.$$

The boundaries of the areas in the regions I to VII are defined in terms of the following coefficients:

$$v_1 = \frac{-\eta_1}{\beta}, v_2 = \frac{-\eta_2}{\beta}, v_3 = \frac{Qy_1 - K_2}{K_1}, v_4 = \frac{Qy_2 - K_4}{K_3}, v_5 = \frac{Qy_1 - K_7}{K_5}$$
$$V_6 = \frac{K}{6}K_5, V_7 = \frac{-K_7}{K_5}, V_8 = V_3 - \frac{K_4 y_1}{K_1 y_2}$$

Let $Z_i(A_i, q^*_i) = q^*_i(A_i\beta + \eta_i - B_i q^*_i\beta)$ with $i = 1, 2$. The expression of the expected terminal value of the production option is given by:

$E^*\{W^{nr}(A_1,A_2,n\tau)\} =$
$\int_{-\infty}^{v2}\int_{v3}^{\infty}Z_1(A_1,Qy_1)f^*(A_1,A_2)dA_1 d_2(IV) + \int_{v2}^{\infty}\int_{v5+V6A2}^{\infty}Z_1(A_1,Qy_1)f^*(A_1,A_2)dA_1 d_2(IV) +$

$$\int_{v2}^{\infty}\int_{-\infty}^{v7+V6A_2} Z_1(A_1, K_1A_1 + K_2) + Z_2(A_2, K_3A_2 + K_4)] + f*(A_1, A_2)dA_1d_2\,(VI)\ +$$

$$\int_{v2}^{\infty}\int_{-\infty}^{v7+V6A_2} Z_1(A_1, K_5A_1 - K_6A_2 + K_7)Z_2(A_2, y_2Q - \frac{K_5A_1 - K_6A_2 + K_7}{yl}f*(A_1, A_2)dA_1d_2\,(VII)\ +$$

$$\int_{v4}^{\infty}\int_{V_7+V_6A_2}^{V_5+V_6A_2}\Big\{ Z_1(A_1, K_5A_1 - K_6A_2 + K_7)Z_2(A_2, y_2Q - \frac{K_5A_1 - K_6A_2 + K_7}{Y_1}f*(A_1, A_2)dA_1dA_2\,(VII)$$

The net present value of manufacturing products is the sum of the present values for current and future production less the discounted stream of fixed costs and the initial outlay. It is given by:

$$V(a_1, a_2, 0) = W0(a_1, a_2, 0) + \sum_{n=1}^{N-1} Wnr((a_1, a_2, 0)) - \frac{CF(Q)(1 - e^{-rT})}{r} - I_0(Q)$$

Where $W^0(a_1, a_2, 0) = Z_1(a_1, q*(0)) + Z_2(a_2, q*2(0))$ corresponds to the value of producing the first period.

Triantis and Hodder (1990) give an example to illustrate the above methodology. They calculate the NPV of manufacturing two products and specify the values of δ_i and μMi for each product. They use the CAPM to determine the μMi. It is possible to extend all their results in the presence of information costs by applying the CAPMI of Merton (1987) instead of the standard CAPM in the determination of the term μMi. A similar analysis can then be used in the presence of information costs.

IV.   THE VALUATION OF REAL OPTIONS WITHIN INFORMATION COSTS IN A DISCRETE-TIME SETTING.

The majority of the papers concerned with the pricing of real assets in a discrete time setting derive from the models for financial options pioneered by Cox, Ross and Rubinstein (1979).

A The valuation of real assets in a simple discrete-time framework
Salkin (1991) extends the basic binomial option pricing methodology to derive a consistent technique for the pricing of real hydrocarbon reserves. We extend this analysis to account for the effect of information costs.

In the classic binomial model of Cox, Ross and Rubinstein (1979), the price of the underlying asset goes up (u) or down (d) with a probability p and (1 -p). The use of this model is based on the presence of a "twin security" which exactly mimics the structure of the project. Consider an investor who can either trade a commodity or invest in a project which supplies the commodity. The use of the dynamics of prices of the commodity must provide a good foundation for the examination of the structure of the cash flows of the project.

By introducing information costs, the probability of an upward movement in the underlying asset price can be shown to be equal to:

$$p = \frac{r - \lambda c - d}{u - d}$$

The price uncertainty is described by a lattice: $S_{i,t} = S_{0,0}\mu^i d^{i-t}$

Where $S_{0,0}$ is the price of the underlying commodity.

Let us denote P_t : the production of a commodity at time t; F_t : the fixed costs of production at time t; V_t : the variable costs of production per unit of commodity at time t; and τ : corporation tax rate on positive cash flows at time t.

These profiles can be used to construct gross revenue, net revenue and post-tax cash flows. Using a lattice of post-tax cash flows, it is possible to calculate the Expected NPV of the project (ENPV). The lattice gross revenue $G_{i,t}$ corresponds to the spot lattice $S_{i,t}$ times the production profile P_t for all time and states t.

$$G_{i,t} = S_{i,t} P_t$$

The net revenue lattice $N_{i,t}$ pre-taxation corresponds to the gross revenue less the cost profiles F_t and V_t:

$$N_{i,t} = G_{i,t} - F_t - P_t V_t$$

The application of a taxation rate to all positive cash flows, gives a lattice that describes the cash flows of the project:

$$\Phi_{i,t} = N_{i,t} \geq 0, N_{i,t} (1 - r)$$
$$\Phi_{i,t} = N_{i,t} < 0, N_{it}$$

The resulting lattice describes the post tax cash flows of the project. The added value to the project resulting from the ability to implement any decision contingent on the cash flows, $\Phi_{i,t}$.

In general, a decision rule is used to decide on the abandonment of a project, the contraction of its scale, the expansion of its scale, or capacity, etc. For example, the decision to abandon is taken when both the post tax cash flows in the current period are negative and the expected future post cash flows from the current time t and state i is negative.

The expected value of all future post tax cash flows from current time t can be calculated by beginning at the end for $T = N$. If we denote by $\psi_{i,t}$ the expected value of all future post tax cash flows for the current time t and state i, then:

$$\psi_{i,t} = \frac{1}{R + \lambda c} \left[p(\psi_{i+t}, t+1 + \phi_{i+1}, t+1) + (1-p)(\psi_i, t+1 + \phi_{i,t+1}) \right]$$

where R refers to one plus the riskless rate of interest.

Now, it is possible to get a structure of cash flows that accounts for the abandonment decision: $\Pi_{i,t} = Max[\Phi_{i,t}; \psi_{i,t}]$.

Repeating this procedure for all states at each period gives the project's value Π_i with the embedded option to abandon the production. The process by which $\Pi_i 0, 0$ is calculated is denoted by:

$$\Pi = F_n (P_t, F_t, V_t, \tau, \sigma, \lambda s, \lambda c, S_{0,0})$$

B.   The generalization of discrete time models

Trigeorgis (1991) proposed a Log-transformed binomial model for the pricing of several complex investment opportunities with embedded real options. The model can be extended to account for information costs. The value of the expected cash flows or the underlying asset V satisfies the following dynamics:

$$\frac{dV}{V} = \alpha dt + \sigma dz$$

Consider the variable $X = \log V$ and $K = \sigma^2 dt$. If we divide the project's life T into N discrete intervals of length τ, then K can be approximated from $\sigma^2 \frac{T}{N}$.

 Within each interval, X moves up by an amount $\Delta X = H$ with probability $(1 - \Pi)$ or down by the same amount $\Delta X = -H$ with probability $(1 - \Pi)$. The mean of the process is $E(dX) = \mu K$; and its variance is $Var(dX) = K$ with $\mu = \frac{(r + \lambda s)}{\sigma^2} - \frac{1}{2}$.

The mean and the variance of the discrete process are:

$$E(\Delta X) = 2 \Pi H - H \quad \text{and} \quad Var(\Delta X) = H^2 - [E(\Delta X)]^2.$$

The discrete time process is consistent with the continuous diffusion process when $2 \pi H - H = \mu K$, with $\mu = \frac{(r + \lambda s)}{\sigma^2} - \frac{1}{2}$ so $\pi = \frac{1}{2}(1 + \frac{\mu K}{H})$ and $H^2 - (\mu K)^2$ so that $H = \sqrt{K + (\mu K)^2}.$

The model can be implemented in four steps. In the first step, the cash flows CF are specified. In the second step, the model determines the following key variables: the time-step:

K from $\frac{\sigma^2 T}{N}$,

The drift μ from $\frac{(r + \lambda s)}{\sigma^2 - 0.5}$,

The state-step H from $\sqrt{K + (\mu K)^2}.$

And the probability Π from $\frac{1}{2}(1 + \frac{\mu K}{H})$.

Let "j" be the integer of time steps (each of length K), i the integer index for the state variable X (for the net number of ups less downs). Let R(i) be the total investment opportunity value (the project plus its embedded options). In the third step, for each state i, the project's values are $V(i) = e^{(X0 + i H)}$.

The total investment opportunity values are given by the terminal condition

$$R(i) = \max[V(i), 0] .$$

The fourth step follows an iterative procedure. Between two periods, the value of the opportunity in the earlier period j at state i, R'(i) is given by:

$$R'(i) = e^{-(r+\lambda c)(\frac{K}{\sigma^2})} [\pi R(i+1) + (1-\pi) R(i-1)]$$

In this setting, the values of the different real options can be calculated by specifying their payoffs. The payoff of the option to switch or abandon for salvage value S is R'= max(R,S). The payoff of the option to expand by e by investing an amount I4 is R'= R+ max (eV - I4,0). The payoff of the option to contract the project scale by c saving an amount I3 is R'= R + max(I'3- cV,0). The payoff of the option to abandon by defaulting on investment I2 is: R'= max(R -I2, 0). The payoff of the option to defer (until next period) is R'= max $(e^{-(r + \lambda c)T} E(R_{j+1}), R_j)$. When a real option is encountered in the backward procedure, then the total opportunity value is revised to reflect the asymmetry introduced by that flexibility or real option. This general procedure can be applied for the valuation of several projects and firms in the presence of information costs.

V.   SUMMARY

This paper develops some results regarding the valuation of the firm and its real options in the presence of information uncertainty. We propose some simple models for the analysis of the investment decision under

Uncertainty and sunk costs. First, we use Merton (1987) model of capital market equilibrium with incomplete information to determine the appropriate rate for the discounting of risky cash flows under incomplete information. This allows the extension of the EVA concept under incomplete information.

Second, we study potential applications of option pricing theory in continuous time for the valuation of simple and complex real options.

Third, we extend the standard analysis for the valuation of flexibility as a complex option within information uncertainty.

Fourth, a general context is proposed for the valuation of real options and the pricing of real assets in a discrete-time setting. Salkin (1991) shows how to apply the Cox, Ross and Rubinstein (1979) model for the valuation of complex capital budgeting decisions. The methodology is applied to a hypothetical case of a marginal natural resource project. The real benefit of this technique arises in its ability to value more realistically situations in which traditional techniques attributed little or no worth. Following the analysis in Salkin (1991), we develop a simple context for the valuation of real options using option pricing techniques in the presence of information Costs. Then, using the Trigeorgis (1991) general Log-transformed binomial model for the pricing of complex investment opportunities, we provide a context for the valuation of these options under incomplete information. It is possible to use the main results in exotic options to value different real options. However, it is important to note that real options can be sometimes more difficult to value in the presence of information costs and a dependency between different real options in the same project.

ENDNOTES

1. For a survey of important results in the standard literature, the reader can refer to Brealey and Myers (1985) and Bellalah (1998). For the valuation in the presence of information costs, we can refer to Bellalah (2001, 2002).
2. These models undermine the theoretical foundation of standard neoclassical investment models and invalidate the net present value criteria in investment choice under uncertainty. For a survey of this literature, the reader can refer to Pindyck (1991), Trigeorgis (1993 a, b, c 1996), Dixit (1995), Luehrman (1997, 1998) and the references in that paper.
3. The formula can be derived using a similar context as that in Roll (1977), Geske (1979), Whaley (1981) and Bellalah (1999). The valuation by duplication technique can be implemented. Consider the following portfolio of options:
    a/ the purchase of a European call c_a having a strike price I and a maturity date T,
    b/ the purchase of a European call c_b with a strike price $S_{cr,t*}$ and a maturity date t*,
    c/ the sale of a European call option c on the option defined in a/ with a strike price ($S_{cr,t*}$ + D - I) and a maturity date (t* - ε).
    The contingent payoff of this portfolio of options is identical to that of an American call. In a perfect market, the absence of costless arbitrage opportunities ensures that the American call value is identical to that of this portfolio. The American call value must be equal to the sum of the three options in the portfolio.
    The option c_a, can be valued using an extension of the Merton's (1973) commodity option formula or the model in Bellalah (1999). The option c_b can be priced using Bellalah (1999) for µla for which the strike price is $S_{cr,t*}$. The option c_c can be priced using an extension of the compound option for µla proposed in Geske (1979). Since the value of the American call is equivalent to the algebraic sum of the three options in the portfolio, we have: $C = c_a + c_b - c_c$.
4. The uses of such curves appear alo in Pindyck (1988) and He and Pindyck (1989).
5. The solutions to the two last equations for two products involve seven types of feasable solutions. The seven regions denoted by Roman numerals and optimal production quantities for the $(n+1)^{th}$ period are:

    I: $\Gamma_1 \neq 0, \Gamma_2 \neq 0, \Gamma_3 = 0; q*_1 = 0, q*_2 = 0$

    II: $\Gamma_1 = 0, \Gamma_2 \neq 0, \Gamma_3 = 0; q_1* = K_1A_1 + K_2, q*_2 = 0$

    III: $\Gamma_1 \neq 0, \Gamma_2 = 0, \Gamma_3 = 0; q_1* = 0, q*_2 = K_3A_2 + K_4$

    IV: $\Gamma_1 = 0, \Gamma_2 \neq 0, \Gamma_3 = 0; q_1* = Qy_1, q_2, q*_2 = 0$

    V: $\Gamma_1 \neq 0, \Gamma_2 = 0, \Gamma_3 \neq 0; q*_1 = 0, q_2* = Qy_1, q_2$

    VI: $\Gamma_1 = 0, \Gamma_2 = 0, \Gamma_3 \neq 0; q_1* = K_1A_1 + K_2, q*_2 = K_3A_2 + K_4$

    VII: $\Gamma_1 = 0, \Gamma_2 = 0, \Gamma_3 \neq 0; q_1* = K_5A_1 - K_6, A_2 + K_7, q*_2 = y_2 [Q - \frac{q_1}{y_1}]$

    where: $K_1 = \frac{1}{2B_1}, K_2 = \frac{\eta_1}{2B_1\beta}, K_3 \frac{1}{2B_2}, K_4 \frac{\eta_2}{2B_2}, K_5 = \frac{1}{2B'}, K_6 = \frac{y_2}{y_1}K_5, K_7 = \frac{K_5}{\beta}\eta_1 - \frac{y_2}{y_1}\eta_2 + \frac{Y^2_2}{y_1}2B_2Q\beta$

    $B' = \left[B_1 + \frac{y^2_2}{y^2_1}B_2 \right]$

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